

Appendix A. Using the SGBI to Process the Constraints

In this appendix, we detail the algorithms to process each of the three different queries presented in this paper using the proposed SGBI. In particular, Algorithm 1 shows how an inside constraint where a granule map has been specified for the reference object, Algorithm 2 details the processing of a constraint where a granule map has been specified for the target objects, and finally, Algorithm 3 processes a constraint where both a particular granularity has been specified for both reference and target objects.

Algorithm 1 Query processing with granule maps for the reference object

Require: Regarding the input: *SGBI* is the spatial index, *GMapNameRef* is the granule map to be used for the reference object, *refObject* is the reference object in the constraint, *R* is the radius of the inside query, and *threshold* is the probability that the objects in the answer set must achieve. It also requires *geom*, an object specialized in performing geometric operations.

Ensure: Retrieves the objects that satisfies the constraint with a probability above the given threshold.

```
1: // candSet is a table where each object is tagged with its probability
2: candSet  $\leftarrow \emptyset$ 
3: candGranules  $\leftarrow$  SGBI.getGranules(GMapNameRef, refObject.area)
4: for g in candGranules do
5:   probRefObjectInG  $\leftarrow$  inGrProb(g, refObject)
6:   if probRefObjectInG > 0 then
7:     extArea  $\leftarrow$  geom.buffer(g.area, R)
8:     affObjects  $\leftarrow$  SGBI.getObjects(extArea)
9:     for obj in affObjects do
10:      probInArea  $\leftarrow$  calculateProbInArea(obj, extArea)
11:      // update the probability by adding the current value
12:      candSet.update(obj, probRefObjectInG * probInArea)
13:     end for
14:   end if
15: end for
16: // filter out the objects above the probability threshold
17: candSet.filterOut(threshold)
18: returns candSet
```

Algorithm 2 Query processing with granule maps for the target objects

Require: Regarding the input: *SGBI* is the spatial index, *GMapNameTgt* is the granule map to be used for the target objects, *refObject* is the reference object in the constraint, *R* is the radius of the inside query, and *threshold* is the probability that the objects in the answer set must achieve. It also requires *geom*, an object specialized in performing geometric operations.

Ensure: Retrieves the objects that satisfies the constraint with a probability above the given threshold.

```
1: /* candSet is a table where each object is tagged with its probability */
2: candSet  $\leftarrow \emptyset$ 
3: refArea  $\leftarrow$  geom.buffer(refObject.area, R)
4: candGranules  $\leftarrow$  SGBI.getGranules(GMapNameTgt, refArea)
5: for g in candGranules do
6:   extArea  $\leftarrow$  geom.buffer(g.area, R)
7:   probGranuleG  $\leftarrow$  calculateProbInArea(refObject, extArea)
8:   if probGranuleG > 0 then
9:     affObjects  $\leftarrow$  SGBI.getObjects(g.area)
10:    for obj in affObjects do
11:      probInGranuleG  $\leftarrow$  inGrProb(g, obj)
12:      // update the probability by adding the current value
13:      candSet.update(obj, probGranuleG * probInGranuleG)
14:    end for
15:  end if
16: end for
17: // filter out the objects above the probability threshold
18: candSet.filterOut(threshold)
19: returns candSet
```

Algorithm 3 Query processing with granule maps for the ref. and the target objects

Require: Regarding the input: *SGBI* is the spatial index, *GMapNameRef* is the granule map to be used for the reference object, *GMapNameTgt* is the granule map to be used for the target objects, *refObject* is the reference object in the constraint, *R* is the radius of the inside query, and *threshold* is the probability that the objects in the answer set must achieve. It also requires *geom*, an object specialized in performing geometric operations.

Ensure: Retrieves the objects that satisfies the constraint with a probability above the given threshold.

```

1: /* candSet is a table where each object is tagged with its probability */
2: candSet  $\leftarrow \emptyset$ 
3: candRefGranules  $\leftarrow$  SGBI.getGranules(GMapNameRef, refObject.UG.area)
4: for g in candRefGranules do
5:   probRefObjectInG  $\leftarrow$  inGrProb(g, refObject)
6:   if probRefObjectInG > 0 then
7:     extArea  $\leftarrow$  geom.buffer(g.area, R)
8:     candTgtGranules  $\leftarrow$  SGBI.getGranules(GMapNameTgt, extArea)
9:     for h in candTgtGranules do
10:      if geom.intersects(extArea, h.area) then
11:        affObjects  $\leftarrow$  SGBI.getObjects(h.area)
12:        for obj in affObjects do
13:          probInGranuleH  $\leftarrow$  inGrProb(h, obj)
14:          // update the probability by adding the current value
15:          candSet.update(obj, probRefObjectInG * probInGranuleH)
16:        end for
17:      end if
18:    end for
19:  end if
20: end for
21: // filter out the objects above the probability threshold
22: candSet.filterOut(threshold)
23: returns candSet

```

Appendix B. Extended Experimental Results

In this appendix we present the whole set of experiments carried out for both probability distributions evaluated in our contribution, namely uniform and normal probability distributions. The experiments evaluate the three types of constraints presented in the paper: granule maps considered for the reference object, for the target objects, and for both (reference and target objects). As in the main paper, they are evaluated over both generated and country-based granule maps. Besides, the influence of each parameter on the performance results is shown both for a single scenario, and for the whole set of scenarios and positions for the reference object.

Appendix B.1. Experimental Results for Generated Granule Maps

Each row in Figures B.1 and B.2 displays the results for each of the three types of constraints using the non-indexed implementation (left column, *No Indexed*) and the results achieved using the SGBI implementation (right column, *Indexed*). To cover the scenario in these tests using different granularities, we have generated a large set of synthetic granule maps in a 4,000 by 4,000 grid using an algorithm based on building Voronoi diagrams from a set of random points. In particular, we have constructed 20 synthetic granule maps which contain 10–70 location granules with a mean of 7.58 edges per granule. Table B.1 shows the parameters used to perform this set of experiments.

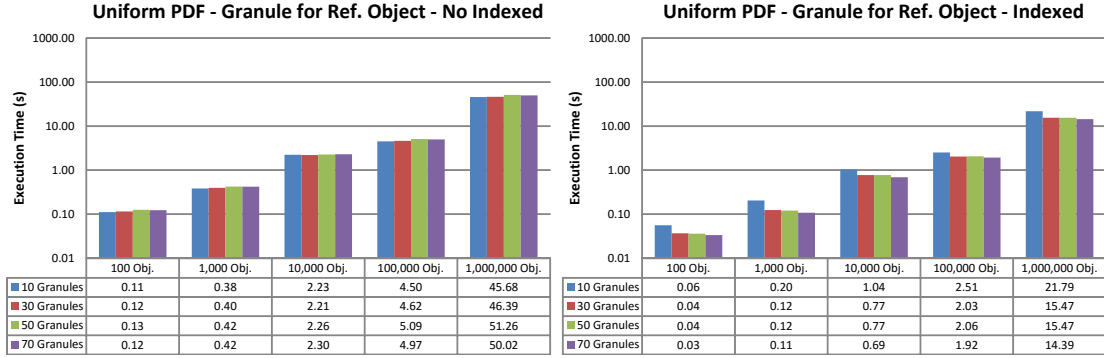
Number of granule maps	20 (5 granule maps for each number of granules)
Number of granules in maps	10, 30, 50, 70
Total target objects in \mathcal{D}	10^2 , 10^3 , 10^4 , 10^5 , 10^6
Locations of the reference object (O_r)	(2000,2000) , $(2000 \pm 1500, 2000 \pm 1500)$
Uniform pdf: Radius for uncertainty area of reference and target objects	10, 30 , 50
Normal pdf: standard deviation σ of reference and target objects	1, 5 , 10
R (inside radius)	100, 300 , 500
p (probability threshold)	0.25, 0.50 , 0.75

Table B.1: Generated scenarios: parameters and their different values for the performance experiments of queries with granule maps (all the possible combinations were evaluated). The bold values are the default values used for the detailed analysis of the influence of the different parameters.

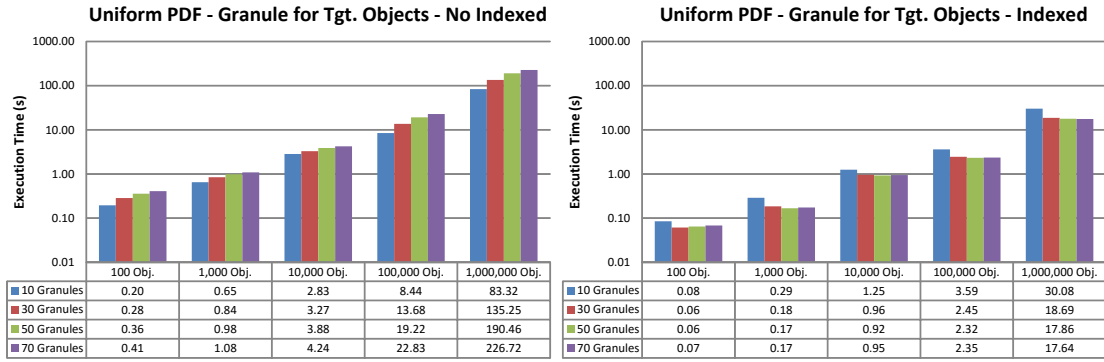
Unsurprisingly, the execution times depend directly on the number of objects in the scenario and on the semantics of the different constraints, as they affect the number of granules involved in the query. In fact, the amount of granules involved in the constraint processing acts as a multiplier (each granule involved implies a complete calculation of its contribution to the final probability of an object to be part of the answer). In the first case (Figures B.1a and B.2a), the amount of affected granules was low, as only the ones that might contain the reference object were involved in the calculations (in the experiments, a mean of 1.47 and 1.56 granules were involved in this kind of queries for the uniform and the normal distributions, respectively).

In the two other cases (Figures B.1b and B.1c for the uniform pdf, and Figures B.2b and B.2c for the normal pdf), as the query semantics became more complex, the costs increased linearly with the amount of granules involved (an average of 8.06 and 13.45 granules for the uniform pdf for the target objects *-tgtObjects-* and for the reference and target objects *-both-* constraints, and an average of 8.81 and 13.73 granules for the normal pdf, respectively)¹. In all

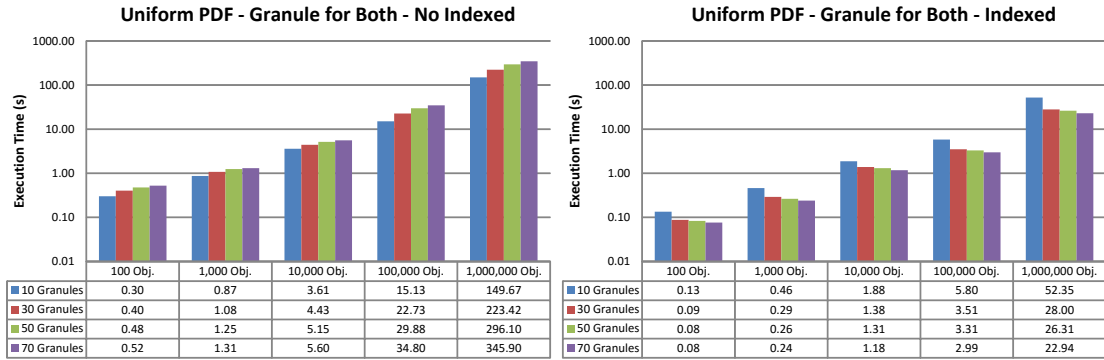
¹Note that they are slightly different due to the difference in the uncertainty area considered. While in the case of the uniform pdf the uncertainty area is directly a disk defined by the uncertainty radius, in the case of the normal pdf it is calculated depending on the sigma



(a) Processing times using a uniform pdf considering granule maps for the reference object.



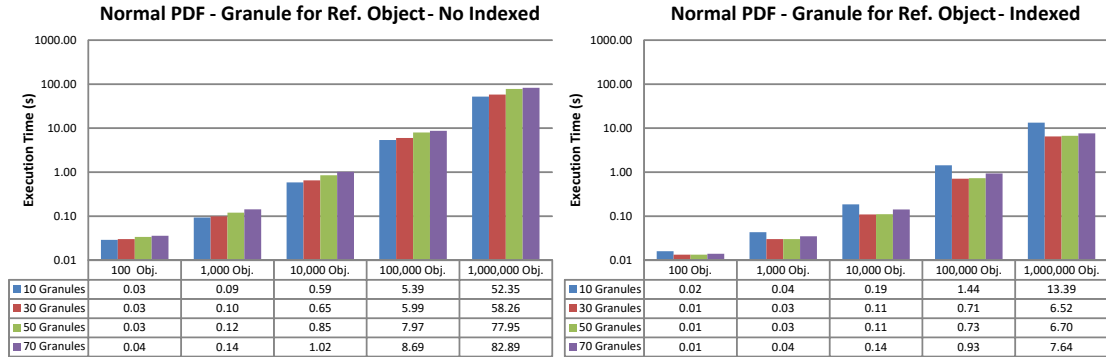
(b) Processing times using a uniform pdf considering granule maps for the target objects.



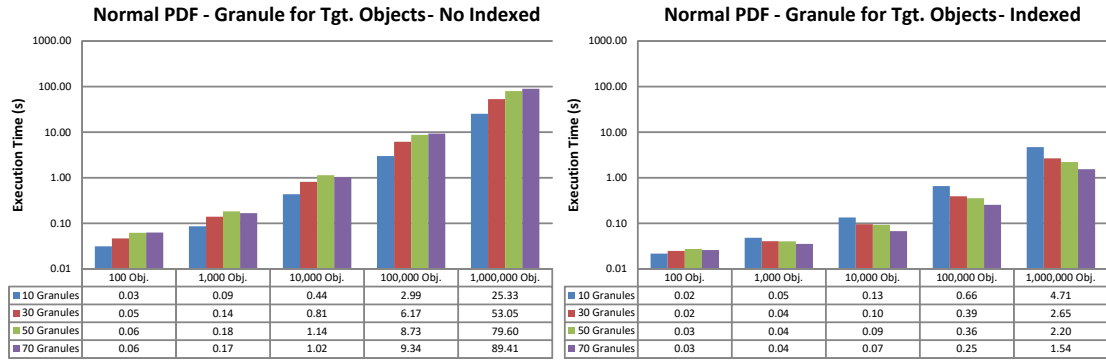
(c) Processing times using a uniform pdf considering granule maps for both the reference and target objects.

Figure B.1: Performance results in the generated scenarios for the uniform pdf. The X axis shows the number of objects in the scenario (ranging from 100 to 1,000,000), while the Y axis (in log scale) shows the query execution time (in seconds). The graphs on the left show the times when no SGBI is used, and the ones on the right when it is used to index the elements in the scenario.

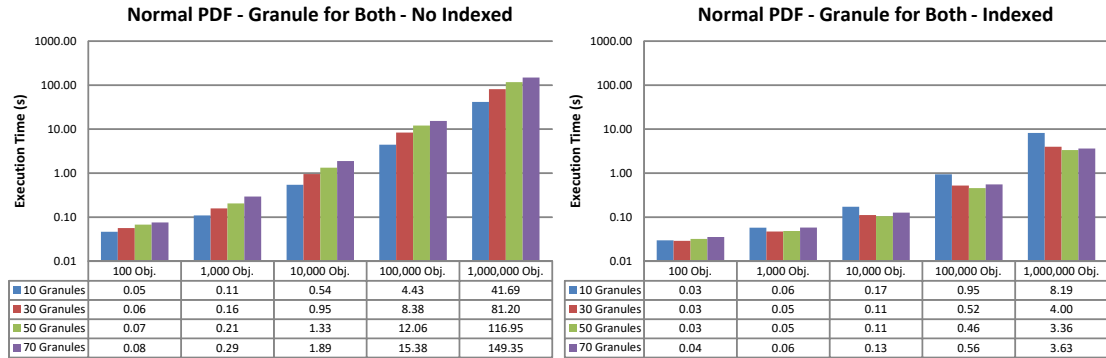
and, thus, while quite similar, the uncertainty areas are not exactly the same.



(a) Processing times using a normal pdf considering granule maps for the reference object.



(b) Processing times using a normal pdf considering granule maps for the target objects.



(c) Processing times using a normal pdf considering granule maps for both the reference and target objects.

Figure B.2: Performance results in the generated scenarios for the normal pdf. The X axis shows the number of objects in the scenario (ranging from 100 to 1,000,000), while the Y axis (in log scale) shows the query execution time (in seconds). The graphs on the left show the times when no SGBI is used, and the ones on the right when it is used to index the elements in the scenario.

the cases, the use of the SGBI improved dramatically the performance of the approach. Note how, being the area of the scenario fixed, as the number of granules increases the area per granule decreases, and this makes the SGBI more efficient when it comes to filtering out object candidates. This can be seen in the right column of Figures B.1 and B.2, where the more granules covering the scenario, the lower the processing times.

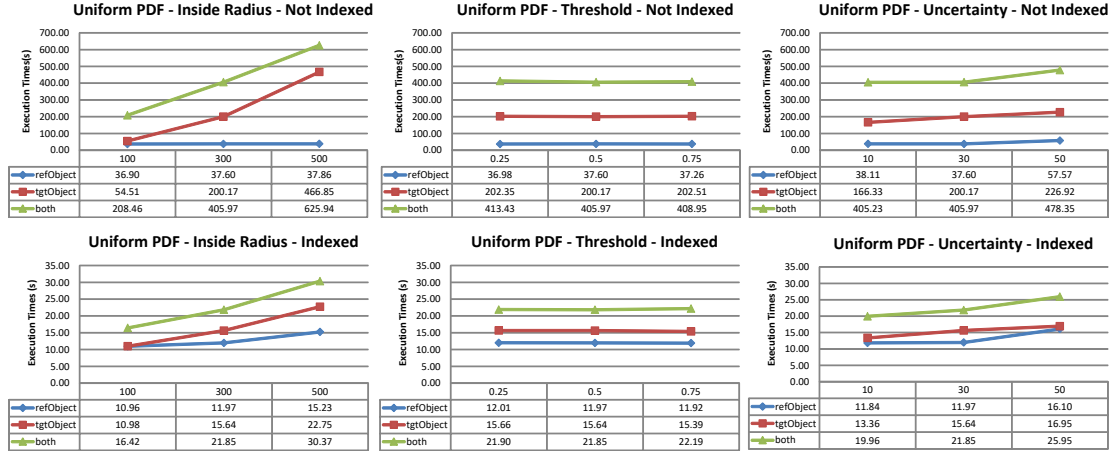
In Figures B.3 and B.4, we show the detailed influence of each of the parameters for uniform and normal pdfs. Figures B.3a and B.4a show the performance for the different parameter sweeps fixing the rest of parameters to the bold values in Table B.1 (the scenario is one of the five available involving a granule map with 70 granules), while Figures B.3b and B.4b show the aggregated values for all the scenarios, in order to show the global trend.

We can see how, when no indexing is used, the inside radius has no influence when we are considering the constraint for the reference object (*refObject*). This is due to the fact that, in such cases, we have to check all the objects of the scenario against each of the granules where the reference object might be; as neither the uncertainty nor its position changes, the number of tests remains the same regardless the inside radius (this does not apply to the other constraints, as the greater the radius, the more granules are involved in the query and the more object tests we have to perform). When the SGBI is used, this no longer applies, as we can filter out non-relevant objects, and the number of filtered objects depends directly on the length of the inside radius (the greater the radius, the less objects are filtered out). In our current implementation, we can see how the threshold applied has no influence on the performance of the approach. Finally, the uncertainty parameter behaves as the inside radius, but its influence is lower in the experiments, as it is smaller than the inside radius values.

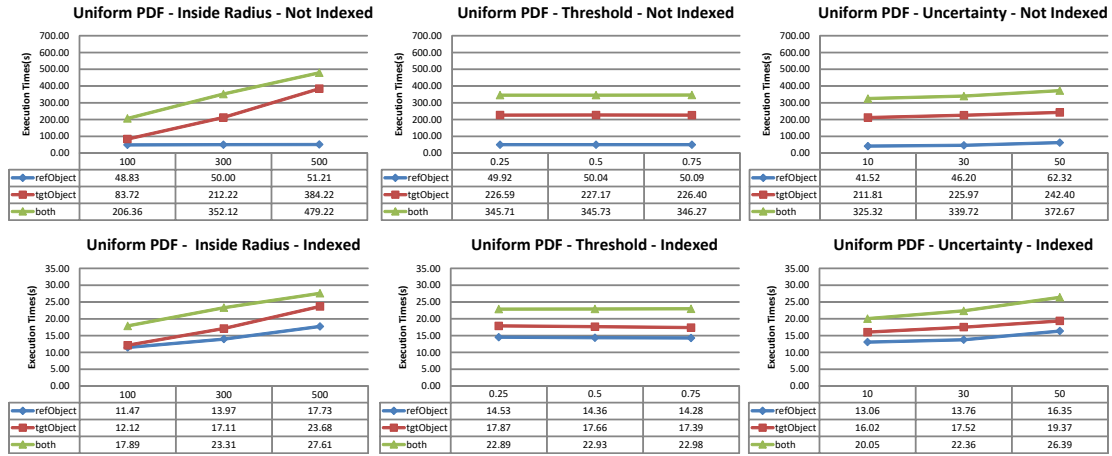
When the SGBI is used, regarding the influence of the inside radius and the uncertainty area, we can see how the larger the inside radius or the uncertainty area of the constraint, the higher the processing costs, as more objects are relevant for the query. Furthermore, with respect to the threshold of the queries, the value of the threshold has no impact on the processing time for each type of constraint, since, in our implementation, we calculate the final probability for all the relevant objects.

Let us also note the difference between the uniform and normal pdf graphs: the *refObject* constraint has the lowest processing time for the uniform pdf, while it has the highest one for the normal pdf, when using the SGBI, or a similar cost to the *tgtObjects* constraint, if the SGBI is not used. This is due to the cost of the calculation of the final probabilities for each object that has not been filtered by the SGBI: it is quadratic regarding the number of edges of the polygons involved for the uniform pdf, and it is linear for the normal pdf. In the case of the *refObject* constraint, the probabilities are calculated against the extended (*buffered*) granules, which introduces a lot of edges (in our implementation, up to 16 for each corner). On the other hand, for the *tgtObjects* and the (*both*) constraints, such probabilities are calculated against the granules of the default granule map, which have an average of 8 edges and 25 edges

for the generated and country-based scenarios, respectively (significantly fewer edges than in the case of the buffered granules). In the case of the normal pdf, despite the fact that we have less relevant objects for the *refObject* constraint, and we have to calculate fewer probabilities than for the *tgtObjects* and *both* constraints, the time cost to calculate the probabilities of the buffered granules with many edges is bigger than the time cost to calculate more probabilities against granules with less edges. In the case of the uniform pdf, as the time complexity of the calculation of the probability is quadratic, this effect is not seen, and the *refObject* constraint is computed faster than in the case of the *tgtObject* and *both* constraints.

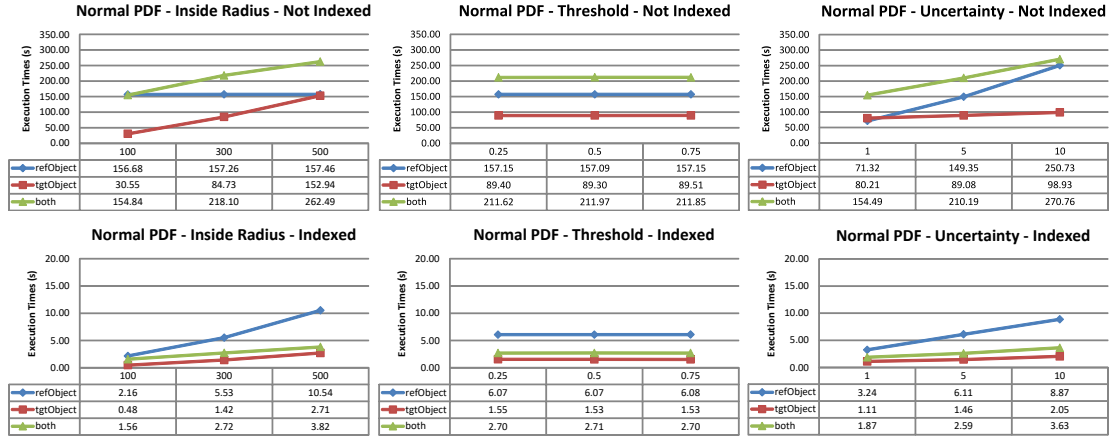


(a) Influence of the different parameters for the uniform pdf in a single scenario.
(generated granule map)

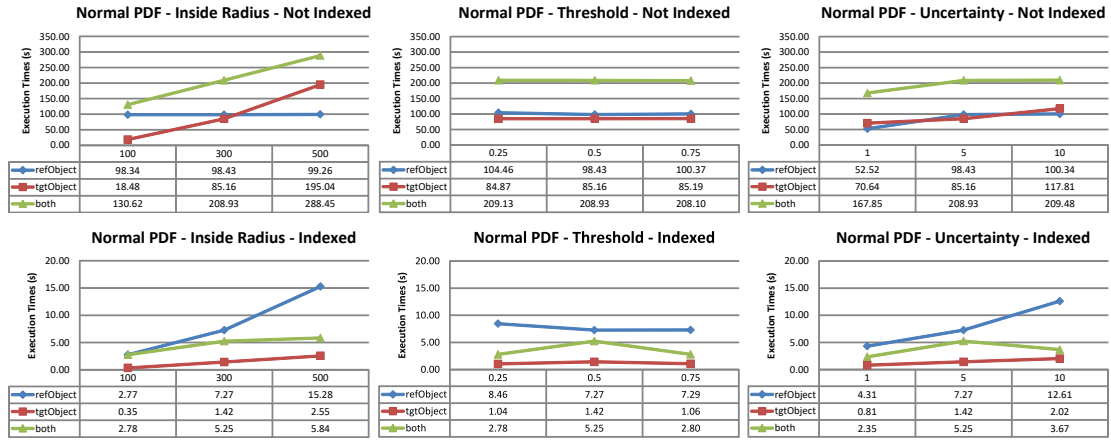


(b) Aggregated influence of the different parameters for the uniform pdf.
(generated granule maps)

Figure B.3: Influence of the different parameters on the processing times of the different constraints considered for the uniform pdf in the case of generated granule maps. The X axis of each graph shows the values for the specific studied parameter (*distance units* for the inside radius, *probability* value for the threshold, and *distance unit* for the uncertainty radius), while the Y axis shows the query execution time (in seconds). The upper rows of the subfigures show the values when no SGBI used, and the lower rows when it is so.



(a) Influence of the different parameters for the normal pdf in a single scenario.
(generated granule maps)



(b) Aggregated influence of the different parameters for the normal pdf.
(generated granule maps)

Figure B.4: Influence of the different parameters on the processing times of the different constraints considered for the normal pdf in the case of generated granule maps. The X axis of each graph shows the values for the specific studied parameter (*distance units* for the inside radius, probability value for the threshold, and σ for the uncertainty), while the Y axis shows the query execution time (in seconds). The upper rows of the subfigures show the values when no SGBI used, and the lower rows when it is so.

Appendix B.2. Experimental Results for Country-Based Granule Maps

In this subsection, we explain the results of the experiments for the Spain and France based scenarios. These sets of granule maps correspond to a lower-detail representation of the maps of Spain and France at two different granularities (*provinces* and *regions*). The corresponding granules were generated by detecting polygons from a vectorial map of Spain and France, with a mean of 15.86 and 25.15 edges per granule at *province* granularity, respectively; and a mean of 10.06 and 55.22 edges per granule at *region granularity*, respectively. These maps are framed within a 4,000 by 4,000 grid, and will be referred to as *country-based granule maps*. Table B.2 shows the parameters used to perform this set of experiments. As the previous set of experiments shown the benefits of using the SGBI, we only performed them using the indexed version to show the feasibility and scalability of our approach.

Number of granule maps	four, two maps (regions and provinces) for each country (Spain and France)
Number of granules in maps	In Spain: 14 (regions map), 47 (provinces map) In France: 22 (regions map), 96 (provinces map)
Total target objects in \mathcal{D}	10^2 , 10^3 , 10^4 , 10^5 , 10^6
Locations of the reference object (O_r)	(2000,2000) , (2000 \pm 500,2000)
Uniform pdf: Radius for uncertainty area of reference and target objects	10, 30 , 50
Normal pdf: standard deviation σ of reference and target objects	1, 5 , 10
R (inside radius)	100, 300 , 500
p (probability threshold)	0.25, 0.50 , 0.75

Table B.2: Country-based scenario: parameters and their different values for the performance experiments of queries with granule maps representing areas of Spain and France (all the possible combinations were evaluated). In these scenarios, we consider the point (2000,2000) as the center of the countries, and a distance unit translates into 0.351 km for the Spanish maps, and into 0.279 km for the French ones. The bold values are the default values used for the detailed analysis of the influence of the different parameters.

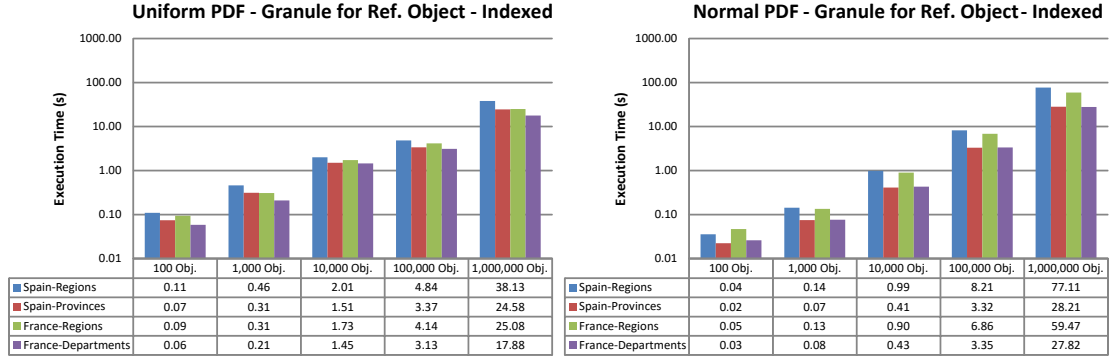
In these scenarios, as explained in the paper, there are several factors that increase the processing times (note the time differences between Figures B.1 and B.2, and Figure B.5):

- Firstly, the number of objects per area unit in these scenarios is higher. This increase in object density leads to a higher number of objects affected by the constraints (in the experiments, about the double than in the first set of experiments).
- Secondly, as we use the same values of radius and the number of granules per area unit is higher, the number of granules affected by each constraint is higher. This, as explained before, acts as a multiplier of the processing costs, as we have to treat each of the affected areas which contributes to the final answer set (e.g., for the most complex constraint, an average of 13.40 and 14.5 granules were affected for each query for the uniform and the normal distributions, respectively).

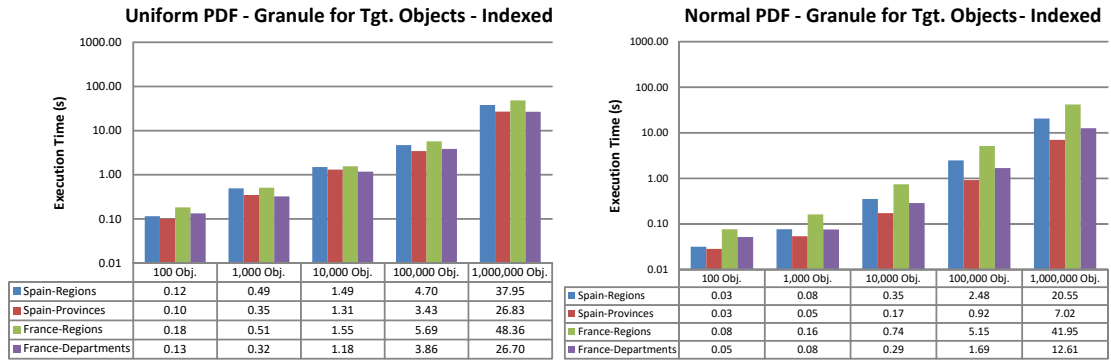
- Finally, we have to bear in mind that the level of detail (the number of edges) of the granules in these scenarios is higher, which also has an impact on the cost of processing both the intersections and the individual object inclusion tests (we move from an average of 7.58 edges per granule in the generated scenarios to an average of 24.11 edges per granule in the latter ones).

We have also noted that, with our implementation, the more objects that there are in the scenario, the costs of the normal pdf grows faster than the uniform pdf calculations (note the how the proportions between equal configurations decreases when the cost of the uniform is initially higher, or increases when otherwise). This leads to a particular situation in Figure B.5c, where, due to the slight differences in the uncertainty areas, the number of affected objects for particular scenarios and settings makes that the cost of normal calculations pdf exceeds the cost of the uniform pdf calculations. However, we want to remark that this is a particular issue about the amount of affected objects in such scenarios, and that it is consistent with all the tendencies exhibited by the rest of the experiments.

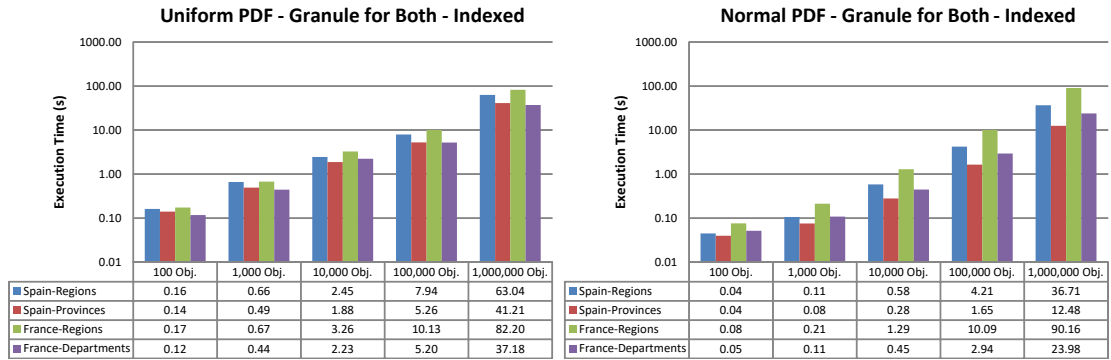
The influence of the parameters can be seen in Figures B.6 and B.7. The analysis of the results are similar than in the case of the generated granule maps in Figures B.3 B.4 of Appendix B.1.



(a) Processing times using a uniform and a normal pdf considering granule maps for the reference object.

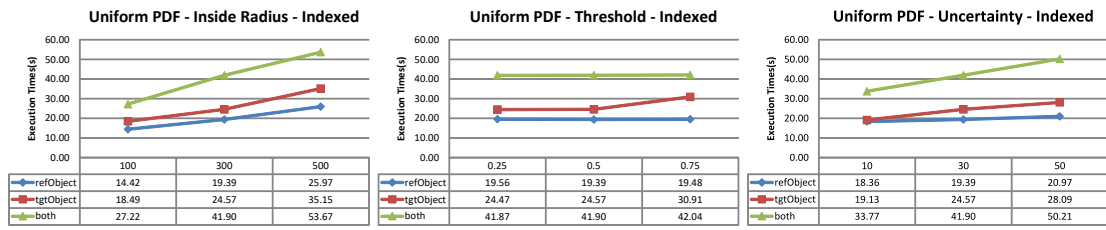


(b) Processing times using a uniform and a normal pdf considering granule maps for the target objects.

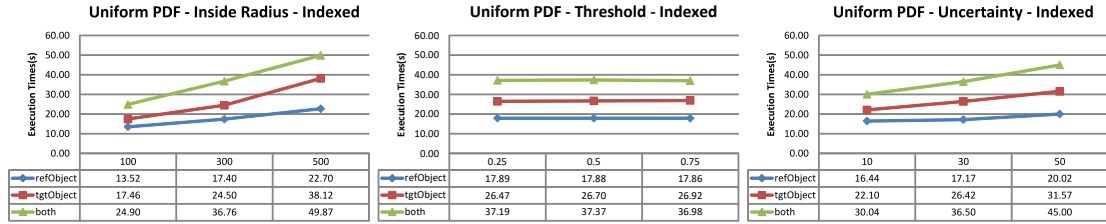


(c) Processing times using a uniform and a normal pdf considering granule maps for both the reference and target objects.

Figure B.5: Performance results in the country-based scenarios for both the uniform and the normal pdfs. The X axis shows the number of objects in the scenario (ranging from 100 to 1,000,000), while the Y axis (in log scale) shows the query execution time (in seconds). Both graphs display the results using the SGBI implementation.

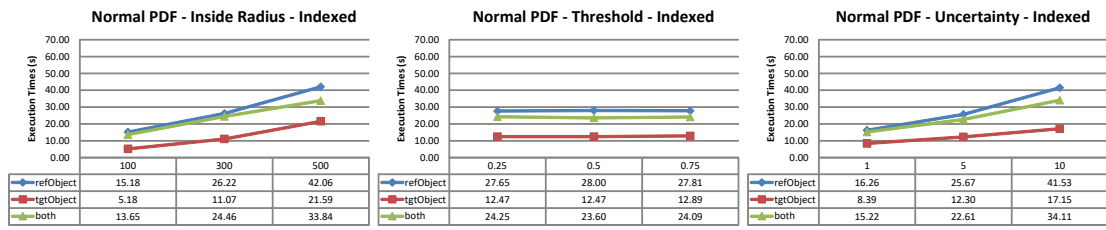


(a) Influence of the different parameters for the uniform pdf in a single scenario.
(country-based granule map)

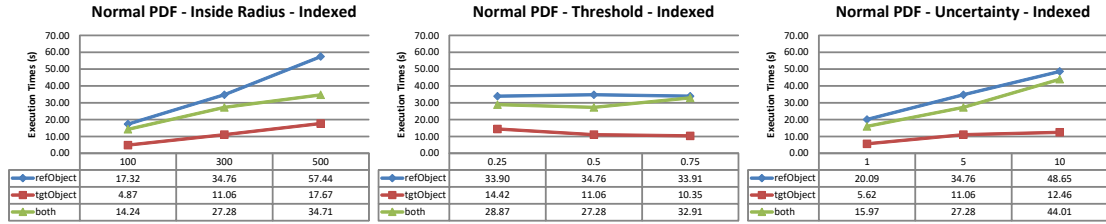


(b) Aggregated influence of the different parameters for the uniform pdf.
(country-based granule maps)

Figure B.6: Influence of the different parameters on the processing times of the different constraints considered for the uniform pdf in the case of country-based granule maps. The X axis of each graph shows the values for the specific studied parameter (*distance units* for the inside radius, probability value for the threshold, and *distance unit* for the uncertainty radius), while the Y axis shows the query execution time (in seconds).



(a) Influence of the different parameters for the normal pdf in a single scenario.
(country-based granule map)



(b) Aggregated influence of the different parameters for the normal pdf.
(country-based granule maps)

Figure B.7: Influence of the different parameters on the processing times of the different constraints considered for the normal pdf in the case of country-based granule maps. The X axis of each graph shows the values for the specific studied parameter (*distance units* for the inside radius, probability value for the threshold, and σ for the uncertainty), while the Y axis shows the query execution time (in seconds).